Local String Field Theory

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Abstract

We consider open bosonic strings. The non-interacting multi-string theory is described by certain free string field operators which we construct. These are shown to have local commutators with respect to a center of mass coordinate. The construction is carried out both in the light cone gauge and in a covariant formulation.

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1 Overview

A classical free open bosonic string in $\mathbb{R}^d$ is specified by a world sheet $X : \mathbb{R} \times [0,\pi] \to \mathbb{R}^d$ which satisfies the wave equation

$$(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2})X^\mu(\tau,\sigma) = 0 \tag{1}$$

with Neumann boundary conditions

$$\frac{\partial X^\mu}{\partial\sigma}(\tau,0) = \frac{\partial X^\mu}{\partial\sigma}(\tau,\pi) = 0 \tag{2}$$

and the constraint

$$\eta_{\mu\nu}(\frac{\partial X^\mu}{\partial\tau} \pm \frac{\partial X^\mu}{\partial\sigma})(\frac{\partial X^\nu}{\partial\tau} \pm \frac{\partial X^\nu}{\partial\sigma}) = 0 \tag{3}$$

where $\eta_{\mu\nu}$ is the Minkowski metric in $\mathbb{R}^d$ with $\eta_{00} = -1, \eta_{kk} = +1$ for $k = 1, \dots, d-1$.

For a single quantized string in the Heisenberg picture one seeks an operator X satisfying canonical commutation relations as well as the field equation and the constraint. There are two standard ways to proceed. On the one hand one can use the constraint to eliminate extra variables and then proceed with canonical quantization. This only works well when carried out in light cone coordinates and is known as the light cone gauge. On the other hand one can quantize directly ignoring the constraint and then impose the constraint by insisting that wave functions be annihilated by constraint operators. This is covariant quantization. We discuss both in more detail below.

In each case we will be able to find a field equation satisfied by the wave functions $\Psi = \Psi(X)$. If string coordinate X is split into a center of mass coordinate x^μ and internal coordinates $X'^\mu = X^\mu - x^\mu$, the wave function can be regarded as a function $\Psi = \Psi(x, X')$. The field equation has the form of a Klein-Gordon equation

$$(-\Box + M^2)\Psi = 0 \tag{4}$$

where $\Box = \eta^{\mu\nu}(\partial/\partial x^\mu)(\partial/\partial x^\nu)$ is the d'Alembertian in the center of mass variable and M^2 is a mass operator which acts on the internal variables X'.

The string field theory describing many strings is obtained by introducing quantized field operators $\Phi = \Phi(x, X')$ obeying $(-\Box + M^2)\Phi = 0$. This

is second quantization. We carry out this construction in both the light cone gauge and the covariant theory. Our main interest is to show that the fields have a vanishing commutator when the center of mass coordinates are spacelike separated. Formally this is

$$[\Phi(x, X'), \Phi(y, Y')] = 0 \qquad \text{if } (x-y)^2 > 0 \tag{5}$$

and we will give a precise version. This report summarizes the results of two papers [1], [2]. Earlier results in the physics literature can be found in [3], [4], [5], [6].

Notation: One can write the two-dimensional wave equation as a first order system

$$\frac{\partial X^\mu}{\partial \tau} = P^\mu \qquad \frac{\partial P^\mu}{\partial \tau} = \frac{\partial^2 X^\mu}{\partial \sigma^2} \tag{6}$$

Suppose we expand in a cosine series as dictated by the boundary conditions. The coefficients are the center of mass coordinates

$$x^\mu(\tau) = \frac{1}{\pi}\int_0^\pi X^\mu(\tau,\sigma)d\sigma \qquad p^\mu(\tau) = \frac{1}{\pi}\int_0^\pi P^\mu(\tau,\sigma)d\sigma \tag{7}$$

and the internal coordinates for $n = 1, 2, \dots$

$$x_n^\mu(\tau) = \frac{\sqrt{2}}{\pi}\int_0^\pi X^\mu(\tau,\sigma)\cos n\sigma d\sigma \qquad p_n^\mu(\tau) = \frac{\sqrt{2}}{\pi}\int_0^\pi P^\mu(\tau,\sigma)\cos n\sigma d\sigma \tag{8}$$

These satisfy

$$\begin{aligned} dx^\mu/d\tau &= p^\mu \qquad & dp^\mu/d\tau &= 0 \\ dx_n^\mu/d\tau &= p_n^\mu \qquad & dp_n^\mu/d\tau &= -n^2 x_n^\mu \end{aligned} \tag{9}$$

2 Light cone gauge

Assuming X satisfies the wave equation, boundary conditions, and constraints, we still have many possibilities for parameterizing the solution. To take advantage of this we change to light cone coordinates defined by mapping $x = (x^0, \dots, x^{d-1})$ to $(x^+, x^-, \tilde{x})$ where

$$x^\pm = (x^0 \pm x^{d-1})/\sqrt{2} \qquad \tilde{x} = (x^1, \dots, x^{d-2}) \tag{10}$$

A solution is said to be in the *light cone gauge* if $X^+(\tau,\sigma) = p^+\tau$. As to the existence of this gauge we have the following:

Lemma 1 *Let $X^\mu(\tau,\sigma)$ satisfy the wave equation, boundary conditions, and constraints and*

$$P^+ = \frac{\partial X^+}{\partial \tau} > 0 \tag{10}$$

Then there exists a conformal diffeomorphism on (τ,σ) such that in the new coordinates all these conditions still hold and in addition $X^+ = p^+\tau$ with $p^+ > 0$

The condition (1) says that τ is a forward moving parameter in a certain sense. For the proof see [1].

Suppose then we have a solution in the light cone gauge. This choice of gauge and the constraints can be used to eliminate $x^+, p^-, x_n^\pm, p_n^\pm$ as dynamical variables. If we take $x^+ = p^+\tau$ as the time parameter we find that the dynamical equations have become

$$\begin{aligned} dx^-/dx^+ &= p^-/p^+ &\qquad dp^+/dx^+ &= 0 \\ dx^k/dx^+ &= p^k/p^+ &\qquad dp^k/dx^+ &= 0 \\ dx_n^k/dx^+ &= p_n^k/p^+ &\qquad dp_n^k/dx^+ &= (-n^2/p^+)x_n^k \end{aligned} \tag{11}$$

This is a Hamiltonian system with Hamiltonian

$$p^- = \frac{1}{2p^+}\left(\tilde{p}^2 + \sum_{n=1}^{\infty}(\tilde{p}_n^2 + n^2\tilde{x}_n^2)\right) \tag{12}$$

Now we quantize this system by imposing canonical commutation relations on the variables $(p^+, x^-), (x^k, p^k), (x_n^k, p_n^k)$. These can be realized as operators on a Hilbert space of Fock valued functions

$$\mathcal{L}^2(\mathbb{R}^+ \times \mathbb{R}^{d-2}, \mathcal{F}, dp^+ d\tilde{p}/2p^+) \tag{13}$$

The operators p^+, p^k are multiplication operators. The operators x_n^k, p_n^k act on the Fock space of transverse modes

$$\mathcal{F} = \mathcal{F}(\mathcal{L}^{2,\perp}([0,\pi], \mathbb{C}^{d-2})) \tag{14}$$

Here $\mathcal{L}^{2,\perp}$ means the subspace of $\mathcal{L}^2$ orthogonal to the constants and

$$x_n^k = (2n)^{-1/2}((a_n^k)^* + a_n^k) \qquad p_n^k = i(n/2)^{1/2}((a_n^k)^* - a_n^k) \tag{15}$$

where $(a_n^k)^*$ is the creation operator for function $(0,\dots,\sqrt{\frac{2}{\pi}}\cos n\sigma,\dots,0)$ (entry in the k^{th} slot).

The quantum Hamiltonian p^- then has the form

$$p^- = \frac{1}{2p^+}\left(\tilde{p}^2 + M^2\right) \tag{16}$$

where

$$\begin{aligned} M^2 &\equiv \sum_{n=1}^{\infty}(:\tilde{p}_n^2: + n^2 :\tilde{x}_n^2:) - 2a \\ &= 2\left(\sum_{n=1}^{\infty}\sum_{k=1}^{d-2} n(a_n^k)^*(a_n^k) - a\right) \\ &= 2\left(\sum_{n=1}^{\infty}\sum_{k=1}^{d-2} \alpha_{-n}^k\alpha_n^k - a\right) \end{aligned} \tag{17}$$

Here we have Wick ordered and allowed an adjustment by an arbitrary constant $2a$. In the last line we have introduced the notation more common in string theory $\alpha_n^k = -i\sqrt{n}a_n^k$ and $\alpha_{-n}^k = i\sqrt{n}(a_n^k)^*$. The operator M^2 on $\mathcal{F}$ is a sum of harmonic oscillators. It is self-adjoint and has spectrum $-2a,\ 2-2a\ ,4-2a\ ,\dots$. The operator p^- is also self adjoint.

In the Schrodinger picture our Fock-valued wave functions evolve according to

$$\Psi(x^+,p^+,\tilde{p}) = e^{-ip^-x^+}\Psi(p^+,\tilde{p}) \tag{18}$$

In configuration space this becomes

$$\Psi(x^+,x^-,\tilde{x}) = (2\pi)^{-(d-1)/2}\int e^{-ip^-x^+-ip^+x^-+i\tilde{p}\tilde{x}}\Psi(p^+,\tilde{p})dp^+d\tilde{p}/2p^+ \tag{19}$$

which satisfies the Klein-Gordon equation

$$(2\partial_+\partial_- - \tilde{\Delta} + M^2)\Psi = 0 \tag{20}$$

This shows that M^2 can be interpreted as a mass operator for the string. This will be our field equation.

One can now ask whether the theory is Lorentz covariant. This is difficult because of the special choices that have been made. Nevertheless it is formally true provided $d = 26$ and $a = 1$. We do not pursue this, but instead turn to the covariant theory where Lorentz covariance comes naturally.

3 The covariant theory

In the covariant theory we seek a quantization without making special choices of coordinates. Now X^μ and $P^\mu = \partial X^\mu/\partial\tau$ are quantized by solving the wave equation with the commutation relations at $\tau = 0$:

$$[X^\mu(\sigma), P^\nu(\sigma')] = i\pi\delta(\sigma - \sigma')\eta^{\mu\nu} \tag{21}$$

We jump right to the solution which is

$$X^\mu(\tau,\sigma) = x^\mu + p^\mu\tau + i\sum_{n\neq 0}\alpha_n^\mu e^{-in\tau}\frac{\cos n\sigma}{n} \tag{22}$$

where the center of mass operators x^μ, p^μ and the internal operators α_n^μ are required to satisfy the commutation relations

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \qquad\qquad [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n}\eta^{\mu\nu} \tag{23}$$

Operators satisfying these relations can be constructed on a Hilbert space of the form

$$\mathcal{L}^2(\mathbb{R}^d, \mathcal{F}, dp) \tag{24}$$

with p^μ as a multiplication operator and the α_n^μ as creation and annihilation operators on the Fock space

$$\mathcal{F} = \mathcal{F}\left(\mathcal{L}^{2,\perp}([0,\pi], \mathbb{C}^d)\right) \tag{25}$$

These spaces have indefinite inner products, for example on $\mathcal{L}^2([0,\pi],\mathbb{C}^d)$ the inner product is

$$< f, g >= \int_0^\pi \eta_{\mu\nu}\overline{f^\mu(\sigma)}g^\nu(\sigma)d\sigma \tag{26}$$

Now consider the constraint equations which we want to impose as a condition on the wave functions. Passing to Fourier components and Wick ordering one finds that the conditions are $L_n\Psi = 0$ for integer n where:

$$\begin{aligned} L_0 &= \frac{1}{2}\,p^2 + \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n \\ L_m &= \alpha_m\cdot p \;+\; \frac{1}{2}\sum_{n\neq m,0}\alpha_{m-n}\cdot\alpha_n \qquad m\neq 0 \end{aligned} \tag{27}$$

We allow a shift in $L_0 \to L_0 - a$, put aside the constraints for $m < 0$ (a standard compromise), and ask that

$$(L_0 - a)\Psi = 0 \qquad L_m\Psi = 0 \qquad m > 0 \tag{28}$$

These constraints cannot be imposed naively since p^2 has continuous spectrum and $\sum_n \alpha_{-n} \cdot \alpha_n$ has discrete spectrum. To work around this we first decompose our Hilbert space as a direct integral

$$\mathcal{L}^2(\mathbb{R}^d, \mathcal{F}, dp) = \int^{\oplus} \mathcal{L}^2(V_r, \mathcal{F}, d\mu_r)\ dr \tag{29}$$

where $V_r = \{p : p^2 + r = 0\}$ is the mass shell and μ_r is the Lorentz invariant measure on V_r. Then L_0 and L_m are decomposable and we have $L_0 - a = \int^{\oplus} \frac{1}{2}(-r + M^2)\ dr$ where M^2 on $\mathcal{F}$ is given by

$$M^2 = 2\sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n - 2a \tag{30}$$

This again has spectrum $-2a, 2 - 2a, \ldots$. To get a nontrivial null space for $L_0 - a$ we pick the values $r = -2a, 2 - 2a, \ldots$ out of the direct integral and form the direct sum

$$\mathcal{H} = \bigoplus_r \mathcal{L}^2(V_r^{(+)}, \mathcal{F}, d\mu_r) \equiv \bigoplus_r \ \mathcal{H}_r \tag{31}$$

Then L_0, L_m act on this space. On a vector $\Psi = \{\Psi_r\}$ the constraints are

$$M^2\Psi_r = r\Psi_r \qquad L_m\Psi_r = 0 \tag{32}$$

which can be satisfied.

Let

$$\mathcal{H}' = \bigoplus_r \mathcal{H}'_r \tag{33}$$

be the subspace of $\mathcal{H}$ satisfying the constraints. We divide out isotropic elements

$$\mathcal{H}'' = \mathcal{H}' \cap (\mathcal{H}')^{\perp} = \bigoplus_r \mathcal{H}''_r \tag{34}$$

which are null vectors, and form the physical Hilbert space

$$\mathcal{H}^{phys} = \mathcal{H}'/\mathcal{H}'' = \bigoplus_r \mathcal{H}^{phys}_r \tag{35}$$

The famous "no-ghost" theorem (see for example [7]) asserts that the inner product is positive definite on $\mathcal{H}^{phys}$ provided $d = 26$ and $a = 1$. We make this choice, so that the sum is over $r = -2, 0, 2, \dots$ The prize for all this is that one now has a natural unitary representation $U(a, \Lambda)$ of the inhomogeneous Lorentz group.

Let us exhibit some physical states. If Ω_0 is the empty state in $\mathcal{F}$ and $f \in \mathcal{L}^2(V_{-2}, \mathbb{C}, d\mu_{-2})$ then

$$\Psi(p) = f(p)\Omega_0 \tag{36}$$

is an element of $\mathcal{H}_{-2}$ It satisfies $M^2\Psi = -2\Psi$ and $L_m\Psi = 0$ and so is an element of in $\mathcal{H}'_{-2} = \mathcal{H}^{phys}_{-2}$. This is a scalar of mass -2 called the *tachyon.* If $f \in \mathcal{L}^2(V_0^+, \mathbb{C}^d, d\mu_0)$ then

$$\Psi(p) = f_\mu(p)\alpha_1^\mu\Omega_0 \tag{37}$$

is an element of $\mathcal{H}_0$. It satisfies $M^2\Psi = 0$ and if $p^\mu f_\mu(p) = 0$ it satisfies $L_m\Psi = 0$ as well and hence is an element of $\mathcal{H}'_0$. Dividing by $\mathcal{H}''_0$ removes longitudinal states with $f_\mu(p) = p_\mu h(p)$ and we get elements of $\mathcal{H}^{phys}_0$. These are identified as photons and this is essentially the Gupta-Bleuler construction. Higher mass physical states can also be exhibited, see [2]

Finally consider a wavefunction $\Psi = \{\Psi_r\}$ with at least the first constraint $M^2\Psi_r = r\Psi_r$ satisfied. Then the Fourier transform

$$\Psi(x) = \sum_r \int_{V_r} e^{-ip\cdot x}\Psi_r(p)d\mu_r(p) \tag{38}$$

again satisfies the Klein-Gordon equation

$$(-\Box + M^2)\Psi = 0 \tag{39}$$

4 String fields

Now we proceed to quantize the two field equations we have identified. Although we have arrived at them in quite different ways they both have the form of a Klein-Gordon equation $(-\Box + M^2)\Phi = 0$ for functions $\Phi : \mathbb{R}^d \to \mathcal{F}$. (We revert to standard coordinates for the light cone gauge). The difference is in the Fock space $\mathcal{F}$. We have

$$\mathcal{F} = \begin{cases} \mathcal{F}\left(\mathcal{L}^{2,\perp}([0,\pi], \mathbb{C}^{d-2})\right) & \text{light cone gauge} \\ \mathcal{F}\left(\mathcal{L}^{2,\perp}([0,\pi], \mathbb{C}^{d})\right) & \text{covariant theory} \end{cases} \tag{40}$$

In the second case there is an indefinite inner product and the remaining constraints $L_m\Phi = 0$ must still be satisfied, The mass operators M^2 on $\mathcal{F}$ are different but have the same spectrum. We discuss the two cases in parallel.

First we consider the "classical" equation, i.e. $(-\Box + M^2)U = 0$ for functions U which are real valued with respect to some conjugation on $\mathcal{F}$. These equations have advanced and retarded fundamental solutions $E^{\pm}$ which are defined on test functions $F \in \mathcal{C}_0^{\infty}(\mathbb{R}^d, \mathcal{F})$. They satisfy $(-\Box + M^2)E^{\pm}F = F$ and $E^{\pm}F$ has support in the causal future/past of the support of F. Explicitly they are given by

$$(E^{\pm}F)(x) = \frac{1}{(2\pi)^{d/2}} \int_{\Gamma_{\pm}\times\mathbb{R}^{d-1}} \frac{e^{ip\cdot x}}{p^2 + M^2}\ \tilde{F}(p)dp \tag{41}$$

where the p^0 contour $\Gamma_{\pm}$ is the real line shifted slightly above/below the real axis. We will also need the propagator function

$$E = E^+ - E^- \tag{42}$$

Then $U = EF$ solves $(-\Box + M^2)U = 0$ and has $\mathcal{C}_0^{\infty}$ Cauchy data on any spacelike hypersurface. We say it is a *regular* solution. Conversely any regular solution U has the form $U = EF$.

If U, V are regular solutions then

$$\sigma(U,V) = \int_{x_0=t} (< U(x), \frac{\partial V}{\partial x^0}(x) > - < \frac{\partial U}{\partial x^0}(x), V(x) >)\ d\vec{x} \tag{43}$$

is independent of t by Green's identity. For definiteness take $t = 0$. Also by Green's identity any regular solution U regarded as a distribution can be expressed in terms of its Cauchy data by

$$< U, F >= \sigma(U, EF) \tag{44}$$

Now we turn to the quantized version. The phase space we want to quantize is the space of Cauchy data for $(-\Box + M^2)U = 0$ on a spacelike hypersurface, or equivalently the space of regular solutions U itself. The form $\sigma(U, V)$ is the natural symplectic form on the space and quantization consists of finding operators $\sigma(\Phi, U)$ indexed by regular solutions $U : \mathbb{R}^d \to \mathcal{F}$ such that

$$[\sigma(\Phi, U), \sigma(\Phi, V)] = -i\sigma(U, V) \tag{45}$$

These are the canonical commutation relations (CCR). Given a representation of the CCR we create a string field operator with test functions $F \in \mathcal{C}_0^\infty(\mathbb{R}^d, \mathcal{F})$ as in (44) by

$$\Phi(F) = \sigma(\Phi, EF) \tag{46}$$

Then Φ satisfies the field equation in the sense of distributions,

$$\Phi((-\Box + M^2)F) = 0 \tag{47}$$

and using $\sigma(EF, EG) = - < F, EG >$ it has the commutator

$$[\Phi(F), \Phi(G)] = -i < F, EG > \tag{48}$$

This is the structure we want, and in fact one can show that any operator valued distribution $\Phi(F)$ satisfying (47),(48) arise from a representation $\sigma(\Phi, F)$ of the CCR in this manner. The locality result is now immediate. If $F, G \in \mathcal{C}_0^\infty(\mathbb{R}^d, \mathcal{F})$ have spacelike separated supports then $< F, EG >= 0$ and hence

$$[\Phi(F), \Phi(G)] = 0 \qquad \text{(locality)} \tag{49}$$

This completes the abstract discussion for the light cone gauge, but for the covariant theory there are still constraints to be satisfied and the interpretation of the constraints seems to depend on the representation.

What representations of the CCR should we consider? What representations might have physical relevance? This is not clear.

If we suppress tachyons there is a distinguished positive energy representation we can consider. The *ad hoc* suppression of tachyons is not really satisfactory, but it does give some insight and makes contact with point field theory. We give some details in the covariant case; the light cone gauge is similar. Excluding $r = -2$ we define as before

$$\mathcal{H}_+ = \bigoplus_{r=0,2,\ldots} \mathcal{H}_r = \bigoplus_{r=0,2,\ldots} \mathcal{L}^2(V_r^+, \mathcal{F}, d\mu_r) \tag{50}$$

For $F \in \mathcal{C}_0^\infty(\mathbb{R}^d, \mathcal{F})$ define $\Pi_+ F \in \mathcal{H}_+$ by

$$(\Pi_+ F)_r(p) = \sqrt{2\pi}\ P_r \tilde{F}(p) \qquad p \in V_r^+ \tag{51}$$

Here P_r is the projection onto the $M^2 = r$ subspace of $\mathcal{F}$. Let a, a^* be creation and annihilation operators on the Fock space $\mathcal{K} = \mathcal{F}(\mathcal{H}_+)$ and define

$$\Phi(F) = a(\Pi_+ F) + a^*(\Pi_+ F) \tag{52}$$

This satisfies (47), (48) and generalizes the positive energy representation for point fields.

Continuing with this positive energy representation we impose the constraint by asking for states annihilated by $L_m \Phi$, actually just the annihilation part of this operator. This turns out to be $\mathcal{K}' = \mathcal{F}(\mathcal{H}'_+)$ and dividing out isotropic elements gives $\mathcal{K}^{phys} = \mathcal{F}(\mathcal{H}_+^{phys})$ which has a positive definite inner product. The field operators $\Phi(F)$ act on $\mathcal{K}^{phys}$ if $\Pi_+ F \in \mathcal{H}'_+$. We call such fields *observable fields*. The observable fields still have the local commutator $[\Phi(F), \Phi(G)] = -i < F, EG >$. However now it is not clear whether this can be made to vanish, i.e. it is not clear whether one can satisfy $\Pi F \in \mathcal{H}'_+$ and still have some control over the support of F. Thus the existence of local observables is not settled in this covariant case; in the light cone gauge there is no problem.

5 Comments

1. There is an interesting extension of these results [1], originally due to Martinec [3]. Consider the light cone gauge and change from a Fock representation for the internal degrees of freedom to a Schrodinger representation. The x_n^k are now independent Gaussian random variables with mean zero and variance $(2n)^{-1}$ and $p_n^k = -i\partial/\partial x_n^k + inx_n^k$. The field equation $(-\Box + M^2)U = 0$ now takes form

$$\left((\frac{\partial}{\partial x^0})^2 - \sum_{k=1}^{d-1} (\frac{\partial}{\partial x^k})^2 - \sum_n \sum_{k=1}^{d-2} \left((\frac{\partial}{\partial x_n^k})^2 - 2n x_n^k \frac{\partial}{\partial x_n^k} \right) - 2 \right) U = 0 \tag{53}$$

Suppose we consider functions $U = U(x^\mu, \{x_n^k\})$ which depend only on a finite number of these modes, and so restrict the sum over n to

$n \leq N$. Then we have a strictly hyperbolic differential equation with domain of dependence defined by the metric

$$-(dx^0)^2 + \sum_{k=1}^{d-1}(dx^k)^2 + \sum_{n=1}^{N}\sum_{k=1}^{d-2}(dx_n^k)^2 \tag{54}$$

Test functions for field operators, formerly Fock valued, can now be regarded as functions $F = F(x^\mu, \{x_n^k\})$ of these variables. Then one can show that if F, G have spacelike separated supports with respect to the above metric then

$$[\Phi(F), \Phi(G)] = 0$$

One says that the field is local with respect to the *string light cone*. This is a limitation on how fast the various modes can grow.

2. Interacting string field theory does not exist, although there are candidates. Is there any chance that such a theory also has a locality property? For some speculation in this direction see [5]

References

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